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INTUITIONISTIC FUZZY MULTIPLICATION OF BV - ALGEBRAS

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Abstract:

In this paper, we introduced the concept of Intuitionistic Fuzzy multiplication to Intuitionistic Fuzzy BV-Algebras. We investigated some of their properties in detail by using the concepts of Intuitionistic fuzzy BV- Ideal and Intuitionistic fuzzy BV-subalgebra.

Key Words: BV-Algebras, Intuitionistic Fuzzy α −multiplication, Intuitionistic fuzzy BV–Ideal, Intuitionistic Fuzzy BV-Subalgebra.

1. Introduction:

L. A. Zadeh was introduced the concept of fuzzy set in 1965 [5]. Several researchers explored at the generalization of the perception of fuzzy subset. Atanassov was introduced the concept "Intuitionistic fuzzy set". Y. Imai and K. Iseki [3] introduced the concept of BCK/BCI algebras in 1978. K.Iseki [4] newly developed the concept of BCI-algebras in 1980. Kim C B and Kim H S [8], developed the new concept of BG-algebras in 2008. T. Priya and T. Ramachandran [6] [7] developed the class of PS-algebras, which is a generalization of BCI/BCK/Q /KU/d algebras. A. Prasanna, M. Premkumar and S.Ismail Mohideen [10], introduced the concept of fuzzy translation and multiplication on B-algebras in 2018. In Ho Hwang, Yong Lin Liu and Hee Sik Kim [6] introduced the concept of BV-Algebras. Tapan Senapati [1], discussed Translations of Intuitionistic Fuzzy B-algebras in 2015. Mohsin Khalid, Rakib Iqbal, Sohail Zafar, Hasan Khalid [2] were introduced Intuitionistic Fuzzy Translation and Multiplication of G-algebra in 2019. In this paper, we introduced the concept of Intuitionistic fuzzy multiplication on Intuitionistic fuzzy BV-subalgebra and Intuitionistic fuzzy BV-ideals of BV- algebras established some of its properties in detail.

2. Preliminaries:

In this section the basic definition of a BV-algebra, fuzzy BV-subalgebra, fuzzy BV-ideal, intuitionistic fuzzy set are recalled.

Definition 2.1: [7, 11]

An algebra $(A, *, 0)$ is said to be a BV-algebra if it satisfies

(i) $x * x = 0,$

(ii) $x * 0 = x,$

(iii) $(x * y) * z = (0 * y) * (z * x)\ for\ all\ x, y, z \in A.$

Example: 2.1.1: [7, 11]

Let $X = \{0, 1, 2, 3\}$ be a set with the following table:

*	0	1	2	3
0	0	1	3	2
1	1	0	2	3
2	2	3	0	1
3	3	2	1	0

Then it is easy to see that $(A, *, 0)$ is a BV-algebra.

Definition 2.2: [10]

A binary relation " $\leq$ " on an BV-algebra A defined as $x \leq y$ if and only if $x * y = 0$.

Definition 2.3: [10]

Let S be non-empty subset of a BV-Algebra A, then S is called a BV–sub-algebra of A, if $x * y \in S\ \forall\ x, y \in S$

Definition 2.4: [6, 11]

Let A be a BV-Algebra and I be a subset of A, then I is called a BV-ideal of A if it satisfies following conditions:

(i) $0 \in I$

(ii) $x * y \in I$ and $y \in I \Longrightarrow x \in I$

Definition 2.5: [6]

Let A be a BV-Algebra. Then a fuzzy set δ in A is defined as $\delta = \{\langle x, \delta(x)\rangle | x \in A\}$, where $\delta(x)$ is known as membership value of x in A and $0 \le \delta(x) \le 1$

Definition 2.6: [1]

Let A be a BV-Algebra. A fuzzy set I in A is called a fuzzy BV-ideal of A if it satisfies following conditions:

(i) $\delta(0) \ge \delta(x)$
(ii) $\delta(x) \ge \min\{\delta(y * x), \delta(y)\}\ \forall x, y \in A$

Definition 2.7: [1]

A fuzzy set δ in a BV-algebra A is called a fuzzy BV-subalgebra of A if $\delta(x * y) \ge min\{\delta(x), \delta(y)\}\ \forall x, y \in A$

Definition 2.8: [1, 2]

An Intuitionistic Fuzzy set A over X is an object having the form $A = \{\langle x, \gamma_A(x), \delta_A(x) | x \in X\rangle\}$ where $\gamma_A : X \to [0,1]$ and $\delta_A : X \to [0,1]$ denoted the degree of membership and the degree of non-membership of x, respectively, with condition $0 \le \gamma_A(x) + \delta_A(x) \le 1 \forall x \in X$.

Definition 2.9 [1,2]

An Intuitionistic Fuzzy set A over X is called an intuitionistic fuzzy subalgebra of X if it satisfies

(i) $\gamma_A(x * y) \ge \min\{\gamma_A(x), \gamma_A(y)\}$
(ii) $\delta_A(x * y) \le \max\{\delta_A(x), \delta_A(y)\}\ \ \forall x, y \in X$

Definition 2.10: [1, 2]

An Intuitionistic Fuzzy set A over X is called an intuitionistic fuzzy ideal of X if it satisfies

(i) $\gamma_A(0) \ge \gamma_A(x), \delta_A(0) \le \delta_A(x),$
(ii) $\gamma_A(x) \ge \min\{\gamma_A(y * x), \gamma_A(y)\}$
(iii) $\delta_A(x) \le \max\{\delta_A(y * x), \delta_A(y)\}\ \ \forall x, y \in X$

3. Intuitionistic Fuzzy Multiplication of BV - Subalgebras and BV - Ideals: [11]

In this section, we introduced the notion of Intuitionistic Fuzzy multiplication of BV-algebras and some theorems on BV- subalgebras. If A denotes a intuitionistic fuzzy BV-algebra of X. An object for any fuzzy set δ of A, we denote $\mathcal{T} = inf\{\delta_A(x) : x \in X\}$.

Definition 3.1:

Let δ be a subset of A and $\alpha \in [0, \mathcal{T}]$. An object having the form $A = \{\langle x, \gamma_A(x), \delta_A(x) | x \in X\rangle\}$ where $\gamma_A : X \to [0,1]$ and $\delta_A : X \to [0,1]$ is said to be an intuitionistic fuzzy-α- multiplication of δ if it satisfies $(\gamma_A)_\alpha^{\mathcal{M}}(x) = \alpha . \gamma_A(x)$ and $(\delta_A)_\alpha^{\mathcal{M}}(x) = \alpha . \delta_A(x)\ \forall x \in A$.

Theorem 3.2:

For any intuitionistic fuzzy BV – algebra δ of A and $\alpha \in [0,1]$, the intuitionistic fuzzy -α – multiplication $A_\alpha^{\mathcal{M}}(x)$ of δ is a fuzzy intuitionistic BV – subalgebra of A.

Proof:

Let $x, y \in A$, and $\alpha \in [0,1]$

Then $(\gamma_A)_\alpha^{\mathcal{M}}(x * y) = \alpha\, \gamma_A(x * y)$
$\ge \alpha \min\{\gamma_A(x), \gamma_A(y)\}$
$= \min\{\alpha . \gamma_A(x), \alpha . \gamma_A(y)\}$
$= \min\{(\gamma_A)_\alpha^{\mathcal{M}}(x), (\gamma_A)_\alpha^{\mathcal{M}}(y)\}$
$(\gamma_A)_\alpha^{\mathcal{M}}(x * y) \ge \min\{(\gamma_A)_\alpha^{\mathcal{M}}(x), (\gamma_A)_\alpha^{\mathcal{M}}(y)\}$

Now $(\delta_A)_\alpha^{\mathcal{M}}(x * y) = \alpha . \delta_A(x * y)$
$\le \alpha . \max\{\delta_A(x), \delta_A(y)\}$
$= \max\{\alpha . \delta_A(x), \alpha . \delta_A(y)\}$
$= \max\{(\delta_A)_\alpha^{\mathcal{M}}(x), (\delta_A)_\alpha^{\mathcal{M}}(y)\}$

This completes the proof.

Theorem 3.3:

Let δ be an intuitionistic fuzzy subset of A such that the intuitionistic fuzzy - α – multiplication $A_\alpha^{\mathcal{M}}(x)$ of δ is an intuitionistic fuzzy subalgebra of A, for some $\alpha \in [0,1]$, then δ is an intuitionistic fuzzy BV – subalgebra of A.

Proof:

Let $x, y \in A$

Assume that $(\gamma_A)_\alpha^{\mathcal{M}}(x)$ of δ is an intuitionistic fuzzy BV – subalgebra of A for $\alpha \in [0,1]$

Then $\alpha . \gamma_A(x * y) = (\gamma_A)_\alpha^{\mathcal{M}}(x * y)$
$\ge \min\{(\gamma_A)_\alpha^{\mathcal{M}}(x), (\gamma_A)_\alpha^{\mathcal{M}}(y)\}$
$= \min\{\alpha . \gamma_A(x), \alpha . \gamma_A(y)\}$
$= \alpha . \min\{\gamma_A(x), \gamma_A(y)\}$
$\gamma_A(x * y) \ge \min\{\gamma_A(x), \gamma_A(y)\}$

Now $\alpha.\delta_A(x*y) = (\delta_A)_\alpha^{\mathcal{M}}(x*y)$
$\leq \max\{(\delta_A)_\alpha^{\mathcal{M}}(x), (\delta_A)_\alpha^{\mathcal{M}}(y)\}$
$= \max\{\alpha.\delta_A(x), \alpha.\delta_A(y)\}$
$\delta_A(x*y) \leq \max\{\delta_A(x), \delta_A(y)\}$

This completes the proof.

Theorem 3.4:

If the intuitionistic fuzzy-α-multiplication $A_\alpha^{\mathcal{M}}(x)$ of δ is an intuitionistic fuzzy BV-ideal of A, then it satisfies the condition $(\gamma_A)_\alpha^{\mathcal{M}}(x*(y*x)) \geq (\gamma_A)_\alpha^{\mathcal{M}}(y)$

Proof:

$(\gamma_A)_\alpha^{\mathcal{M}}(x*(y*x)) = \alpha.(\gamma_A)(x*(y*x))$
$\geq \min\{\alpha.(\gamma_A)\left(y*(x*(y*x))\right), \alpha.((\gamma_A)(y))\}$
$\geq \min\{\alpha.(\gamma_A)(0), \alpha.(\gamma_A)(y)\}$
$\geq \min\{(\gamma_A)_\alpha^{\mathcal{M}}(0), (\gamma_A)_\alpha^{\mathcal{M}}(y)\}$
$(\gamma_A)_\alpha^{\mathcal{M}}(x*(y*x)) \geq (\gamma_A)_\alpha^{\mathcal{M}}(y)$

Theorem 3.5:

If δ is an intuitionistic fuzzy BV-ideal of A, then the intuitionistic fuzzy-α-multiplication $A_\alpha^{\mathcal{M}}(x)$ of δ is an intuitionistic fuzzy BV-ideal of A, for all $\alpha \in [0,1]$

Proof:

Let δ is an fuzzy BV-ideal of A and let $\alpha \in [0,1]$

Then (i) $(\gamma_A)_\alpha^{\mathcal{M}}(0) = \alpha.(\gamma_A)(0) \geq \alpha.(\gamma_A)(x) = (\gamma_A)_\alpha^{\mathcal{M}}(x)$
$\Longrightarrow \gamma_A(0) \geq \gamma_A(x)$

And $(\delta_A)_\alpha^{\mathcal{M}}(0) = \alpha.(\delta_A(0) \leq \alpha.(\delta_A)(x) = (\delta_A)_\alpha^{\mathcal{M}}(x)$
$\Longrightarrow \delta_A(0) \leq \delta_A(x)$

(ii) $(\gamma_A)_\alpha^{\mathcal{M}}(x) = \alpha.(\gamma_A)(x)$
$\geq \alpha.\min\{(\gamma_A)(y*x), (\gamma_A)(y)\}$
$\geq \min\{\alpha.(\gamma_A)(y*x), \alpha.(\gamma_A)(y)\}$
$\geq \min\{(\gamma_A)_\alpha^{\mathcal{M}}(y*x), (\gamma_A)_\alpha^{\mathcal{M}}(y)\}$
$\Longrightarrow (\gamma_A)(x) \geq \min\{(\gamma_A)(y*x), (\gamma_A)(y)\}$

And $(\delta_A)_\alpha^{\mathcal{M}}(x) = \alpha.(\delta_A)(x)$
$\leq \alpha.\max\{(\delta_A)(y*x), (\delta_A)(y)\}$
$\leq \max\{\alpha.(\delta_A)(y*x), \alpha.(\delta_A)(y)\}$
$\leq \max\{(\delta_A)_\alpha^{\mathcal{M}}(y*x), (\delta_A)_\alpha^{\mathcal{M}}(y)\}$
$\Longrightarrow (\delta_A)(x) \leq \max\{(\delta_A)(y*x), (\delta_A)(y)\}$

Hence the intuitionistic fuzzy-α-multiplication $A_\alpha^{\mathcal{M}}(x)$ of δ is an intuitionistic fuzzy BV-ideal of A, for all $\alpha \in [0,1]$

Theorem 3.6:

Let δ be an intuitionistic fuzzy subset of A such that the intuitionistic fuzzy-α-multiplication $A_\alpha^{\mathcal{M}}(x)$ of δ is an intuitionistic fuzzy BV-ideal of A, for some $\alpha \in [0,1]$, then δ is an intuitionistic fuzzy BV-ideal of A.

Proof:

Assume that the intuitionistic fuzzy-α-multiplication $A_\alpha^{\mathcal{M}}(x)$ of δ is an intuitionistic fuzzy BV-ideal of A, for some $\alpha \in [0,1]$. Let $x, y \in A$.

Then (i) $\alpha.(\gamma_A)(0) = (\gamma_A)_\alpha^{\mathcal{M}}(0) \geq (\gamma_A)_\alpha^{\mathcal{M}}(x) = \alpha.(\gamma_A)(x)$
$\Longrightarrow (\gamma_A)(0) \geq (\gamma_A)(x)$

And $\alpha.(\delta_A)(0) = (\delta_A)_\alpha^{\mathcal{M}}(0) \leq (\delta_A)_\alpha^{\mathcal{M}}(x) = \alpha.(\delta_A)(x)$
$\Longrightarrow \delta_A(0) \leq \delta_A(x)$

(ii) $\alpha.(\gamma_A)(x) = (\gamma_A)_\alpha^{\mathcal{M}}(x)$
$\geq \min\{(\gamma_A)_\alpha^{\mathcal{M}}(y*x), (\gamma_A)_\alpha^{\mathcal{M}}(y)\}$
$\geq \min\{\alpha.(\gamma_A)(y*x), \alpha.(\gamma_A)(y)$
$\geq \alpha.\min\{(\gamma_A)(y*x), (\gamma_A)(y)\}$
$(\gamma_A)(x) \geq \min\{(\gamma_A)(y*x), (\gamma_A)(y)\}$

And $\alpha.(\delta_A)(x) = (\delta_A)_\alpha^{\mathcal{M}}(x)$
$\leq \max\{(\delta_A)_\alpha^{\mathcal{M}}(y*x), (\delta_A)_\alpha^{\mathcal{M}}(y)\}$
$\leq \max\{\alpha.(\delta_A)(y*x), \alpha.(\delta_A)(y)\}$
$\leq \alpha.\max\{(\delta_A)(y*x), (\delta_A)(y)\}$
$(\delta_A)(x) \leq \max\{(\delta_A)(y*x), (\delta_A)s(y)\}$

Hence δ is an intuitionistic fuzzy BV-ideal of A.

4. Conclusion:

In this paper we have introduced the notion of intuitionistic fuzzy multiplication on BV-algebras through intuitionistic fuzzy BV-ideals. BV-algebras as another generalization of BCK/BCI/d/Q-Algebras. This concept can further be generalized to n-generated fuzzy sets, bipolar fuzzy set for new results in our future work.

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